

Ex. 2: Latitude, Longitude, Spherical Distance

Geographic Coordinates, Spherical Distance

Geographic coordinates are any of a variety of systems that establishing absolute positions on the surface of the earth. The first globally applicable system that was developed is the simplest to learn: latitude and longitude. **Latitude** is the angular separation between a point and the equator following a great circle. **Longitude** is the angular separation between a point and a defined prime meridian following a latitude circle. (Longitude could not be accurately measured until the 19th century and the prime meridian in England was agreed to in 1884, but ancient cultures were using a system like this nearly 3000 years ago.)

Latitude ranges from 90°S (South Pole) through 90°N (North Pole). Longitude ranges from 180°E to 180°W, both of which are approximate ways of referring to the International Date Line. In many computer applications, use of the directional letters get in the way, so we use positive and negative numbers instead. If we indicate S latitudes and W longitudes with negative numbers, we get a coordinate system that behaves like a conventional Cartesian coordinate system, with minus signs on the bottom half of the vertical coordinate and the left half of the horizontal coordinates.

A **degree** of latitude is about 110 km or 70 miles, so we need to subdivide the degrees to make accurate positions. The traditional means of subdividing angles is to define **minutes** as 1/60 degrees and then **seconds** as 1/60 minutes – a system also dating back millennia and highly inconvenient for computers. The first thing to do with a latitude or longitude expressed that way is to transform it to **decimal degrees**. If a latitude is expressed in degrees-minutes-seconds (DMS) as $d^{\circ} m' s''$, then the decimal degree form DD is

$$DD = d + m/60 + s / 3600 \quad (1)$$

For example, the Washington Monument is at 38°53'22.08377"N 77°2'6.86378" W. Conversions to decimal degrees are

$$\begin{aligned} \text{Latitude:} & \quad 38 + 53/60 + 22.08377/3600 = 38.8894677^{\circ} \\ \text{Longitude:} & \quad -(77 + 2/60 + 6.86378/3600) = -77.03518994^{\circ} \end{aligned}$$

Note the application of the minus sign to the entire calculation in west longitude, not just to the degrees.

Calculating distance on a sphere.

To calculate distances between points identified by latitude and longitude, we use some simple formulas from spherical trigonometry. these calculations are not perfect because Earth is not perfectly spherical.

On a sphere, the closest distance between points follows a **great circle**, which represents the intersection of the sphere with a plane that passes through its center. If the sphere has radius R , so will all its great circles; so the length of an arc is exactly R times the **central angle** measured in **radians**. The central angle is the angle between two points as seen from the center of the earth.

For points on a meridian (line of constant longitude), calculating distance is easy. For example, The Washington Monument is almost directly north of Emerald Island Inlet off the Coast of North Carolina, which has a latitude of $34^{\circ}38'47.31''\text{N}$. Converting Emerald Island Inlet's location to decimal degrees gives $34 + 38/60 + 47.31/3600 = 34.64648^{\circ}$. Subtracting that from the latitude of the Washington Monument (38.8894677°) gives a latitude difference of 4.24300° , and because they are on the same meridian that is a central angle (all meridians are also great circles).

The central angle in radians is $4.24300^{\circ} \times \pi/180 = 0.074054$ radians. (2π radians is a complete circle, which is also 360° , so converting degrees to radians requires multiplying by $\pi/180$.) Multiplying that by the radius of the earth (the average radius of the earth is about 6371 km, 3959 statute miles, or 3440 nautical miles), gives a distance of 293 statute miles or 472 km.

For points on a parallel (line of constant latitude), calculating distance is only slightly more complicated, because we would multiply the radius of the earth by the cosine of the latitude, which gives us the radius from the spin axis rather than from the center of the earth.

Calculating the distance between two points on a sphere starts by determining the central angle on the great circle that passes through both of them. The **spherical law of cosines** is used. This is derived by setting up a triangle consisting of two points and the North Pole, but we will just use the formula.

Call the geographic coordinates of the first point ($lat1, long1$) and the geographic coordinates of the second point ($lat2, long2$), then the angle A between them along a great circle is calculated from

$$\cos(A) = \sin(lat1) \sin(lat2) + \cos(lat1) \cos(lat2) \cos(long1 - long2). \quad (2)$$

As an example, we already calculated the decimal degrees form of the latitude and longitude of the Washington Monument. Another interesting landmark is the position of the old courthouse in New Castle, Delaware, which was used as a center point for drawing the northern boundary of Delaware on a 12 mile radius: latitude 39.664722° , longitude -75.565278° . The distance between the two landmarks can be calculated as follows:

$$\begin{aligned} \cos(A) &= \sin(39.664722^{\circ}) \times \sin(38.8894677^{\circ}) \\ &\quad + \cos(39.664722^{\circ}) \times \cos(38.8894677^{\circ}) \\ &\quad \times \cos[-75.565278^{\circ} - (-77.03518994^{\circ})] \\ &= 0.99971129 \\ A &= \cos^{-1}(0.99971129) = 1.3768195^{\circ} \\ 1.3768195^{\circ} \times \pi / 180 &= 0.024030033 \text{ radians} \\ 0.024030033 \text{ radians} \times 3959 \text{ statute miles} &= 95.1 \text{ miles} \end{aligned}$$

The exercise

Part I. Collect geographic data. ~~Working in groups, use hand-held GPS receivers to collect coordinates in DMS for the front doors of Pearson Hall and for at least 10 coffee shops. You can find enough within the few blocks of Main Street between the Newark Shopping center and Amstel Avenue. Make sure to record seconds of latitude and longitude with as many decimal places as provided by the GPS unit.~~

Part II. Create a data file. ~~Start a new text file called `coffee.txt` in Emacs. Save the file on a directory in a place that you can save or take with you — preferably on a portable USB drive. Enter your data in the following form:~~

The above instructions are crossed out because this field day became a waste of a class period when everyone has a GPS on their phone. However, the data you're going to use were accumulated by previous classes following those instructions. From the class website, obtain the file `coffee.txt`, which has data lines like this:

```
Starbucks, 40, 34, 12.21, -76, -23, -10.55
Daily Grind, 40, 36, 10.45, -76, -24, -14.01
```

The first line of the file is the location in the front of Pearson Hall, in the same format as shown above for the coffee shops. Each line has seven items, the first of which is the name of the coffee shop. The second through seventh columns are integers representing the degrees, minutes, and seconds of the latitude and longitude readings from each location. Notice that the minus sign recorded for the degrees of longitude also applies to the minutes and seconds.

Part III. Using Python, convert the DMS data to DD. The initial program will be modified further in Part V before being handed in.

- 1) Open (in Emacs) a new Python file (`lastname_ex2.py` with comments at the top identifying yourself and the exercise, as before). This should be in the same directory as the `coffee.txt` data file.
- 2) Attach the data file you just created to a Python file object using the `open` function, with an attribute that it can only be read. Put the lines into a list using the `readlines` function. This can be combined in single step.
- 3) Create a `for` loop to iterate through each line in your file, dealing with the data one coffee shop at a time. For each line of the data file:
 - a) `split` the data in each line into the degrees, minutes, seconds components, making sure to convert them to floating-point numbers.
 - b) Convert the latitude and longitude of each shop into decimal degrees. You will need to split the degrees, minutes, and seconds into three variables, and then use Equation (1) of the Spherical Distance writeup.
 - c) `print` your results to the screen, including the name of each shop and the latitude and longitude in decimal degrees.

Common error: messages that an index of a `split` operation is out of range usually mean one of two things: you are missing a comma in the data file, or you have blank lines in the data file. For example, a reference to `split(",") [6]` requires that you have at least 6 commas, separating 7 items, on a line. Blank lines do not have commas.

Part IV. Calculate the distance from Pearson Hall to all of the other coffee shops. Resume editing your `lastname_ex2.py` file (you may wish save a copy under a different filename for safety). The code down to the point where you printed to the screen will still be used, but you need more calculations before you print.

First, add a decision after calculating each latitude/longitude pair in decimal degrees. You will need to be able to calculate an angular difference between two points using Equation (2) of the Spherical Distance discussion above. For this you will need (lat,long) pairs *in radians* for Pearson Hall and for another coffee shop, so that one location serves as ($lat1, long1$) in Equation (2) and the other serves as ($lat2, long2$). You will need a **if** test that determines you are in the first pass (index 0) through the loop to save the location of Pearson Hall for all subsequent calculations.

Do *not* hard-code the location of Pearson Hall into this program -- imagine that the same program could be made to work for geologists working with Penny Hall as their central location. (Further imagine that the grading process may use a different input data file.)

For each coffee shop, once you have determined the great-circle angle A from Equation (2), you should convert it to a distance by multiplying by the radius of the earth. Then, print out the name of the coffee shop and the two distances to the screen for each coffee shop.

Remember that for Python trigonometry, you need to import the math library to obtain the trigonometric functions (and π), and the trigonometric functions only work on *radians*, not degrees.

Checking your work. It would be a good idea to do one of the calculations for one of the coffee shops using a calculator and following the path of the example at the end of the Spherical Distance handout. That is a good way to catch degree/radian problems. Also, including Pearson Hall as both a starting point and a coffee shop is a good test, because the distance should be zero. Finally: check for reasonableness: you know that none of these coffee shops are a full mile away and all of them are more than 50 feet away, so none should be outside the range 0.01 to 1.0 miles. Finally, you know where most of these are – do the distances make sense to you in terms of which ones are closer or farther?

Part V. Questions about accuracy and precision. The official U.S. Government statement of GPS accuracy is that coordinates are accurate with 4 m most of the time and the 95% confidence level is 7.8 m.

1. Some of the GPS units read out to the nearest 0.01 seconds of arc. What is the distance represented on the ground by 0.01 seconds in longitude at the latitude of Newark?
2. What is the distance represented on the ground by 0.01 seconds in latitude?
3. About how many seconds of arc in longitude at Newark represents the typical accuracy of the GPS unit?

Handins for this exercise:

- the Python program created in Part IV, as an attachments to an email. Name your Python program *lastname_ex2.py* with name and exercise number comments inside as before.
- Answer the questions in Part V in the body of the same email you use to submit the attachments.

10 points.

Bonus point:

A maximum of one additional point per person can be obtained by either (1) adding a coffee shop in a similar distance range (please don't include College Square, Suburban Plaza, or south of the Amtrak line). including providing its latitude and longitude as shown in *coffee.txt*, or (2) identifying a coffee shop/restaurant included in this year's list that no longer exists or has changed name. Add this information to the body of the email where you're answering questions — don't modify the *coffee.txt* file for this year.